

**Solutions to the Review Questions at the End of Chapter 1**

1. This question simply involves plugging the appropriate values of *x* into the formulae

(a) *x*=0, *f*(0) = 3×(02) – 4×0 + 2 = 2.

*x*=2, *f*(2) = 3×(22) – 4×2 + 2 = 6.

*x*=-1, *f*(–1) = 3×(–1)2 – 4×(–1) + 2 = 9.

(b) *x*=0, *f*(0) =4×(02) + (2×0) – 3 = –3.

*x*=3, *f*(3) =4×(32) + (2×3) – 3 = 39.

*x*=*a*, *f*(*a*) =4×(*a*2) + (2×*a*) – 3 = 4*a*2+2*a*–3.

*x*=(3+*a*), *f*(3+*a*) =4×((3+*a*)2) + (2×(3+*a*)) – 3 = 4×(9+6*a*+*a*2)+6+2*a*-3=4*a*2+26*a*+39.

(c) In general, the answer is no. We can see from the final example compared with the sum of the previous two in part (b) above that the rule does not follow when *f* is a quadratic function – in other words, *f*(3) + *f(a*) ≠ *f*(3+*a*).

2. To answer this question, we simply need to use the rules for manipulating powers so that we group like terms together

(a) 4*x*5 × 6*x*3 = 24*x*8.

(b) 3*x*2 × 4*y*2 × 8*x*4 × -2*y*4 = –192*x*6*y*6.

(c) (4*p*2*q*3)3 =43*p*2×3*q*3×3 = 64*p*6*q*9.

(d) 6*x*5÷3*x*2 = (6/3)*x*5-2 = 2*x*3.

(e) 7*y*2÷2*y*5 = (7/2)y2-5 = 7/(2y3).

(f) .

(g) (*xy*)3 ÷ *x*3*y*3 = *x*3*y*3 ÷ *x*3*y*3 = 1.

(h) (*xy*)3 - *x*3*y*3 = *x*3*y*3 - *x*3*y*3 = 0.

3. Again, here we use the rules for manipulating powers, but now the powers are not integers and thus effectively involve taking roots

(a) 1251/3 = .

(b) 641/3 = .

(c) 641/4 = .

(d) 93/2 = .

(e) 92/3 = (to 2 decimal places).

(f) 811/2 + 641/2 + 641/3 = .

4. This question involves doing the reverse of the steps in the previous one

(a) 9 = 33.

(b) 625 = 54.

(c) 125-1 = 5-3.

5. Answering this question involves simply rearranging the expression so that the terms in *x* are gathered together on one side of the equation and all of the number terms are on the other side

(a) 3*x*-6 = 6*x* –12, 3*x* = 6, *x* = 2.

(b) 2*x*–304*x* + 8 = *x* + 9–3*x*+4, –302*x* = –2*x*+5, 300*x* = –5, *x* = –5/300 = 0.017 (to 3 decimal places).

(c) , 3(*x*+3) = 2(2*x*-6), 3*x*+9 = 4*x*–12, *x*-3 = 0, *x* = 3.

6. This question involves using the rules of sigma and pi notation to list and simplify all of the terms in the expressions

(a)  = 1+2+3 = 6.

(b) .

(c) (with *n* = 4 and *x* = 3) = .

(d) (with *x* = 2) = .

(e) .

7. Recall that the equation for a straight line is *y* = *a* + *bx* where *a* is the intercept and *b* is the gradient. In the first three cases, the answers arise naturally by plugging the appropriate values directly into this formula, but the others require more thought

1. *y* = –1 + 3*x.*
2. *y* = 4 – 2*x.*
3. *y* = 3 + ½*x.*
4. Care is needed here. If the line crosses the *x*-axis at 3, this is not the intercept. To get the intercept, we need to solve y = *a* + (1/2)*x* for *a* with *y* = 0 and *x* = 3, which gives 0 = *a* + (1/2)×3; so *a* = –2/3 and thus the line is *y* = –2/3 + (1/2)*x*.
5. We first need to find the gradient by solving *y* = *a* + *bx* for 2, *x* = 3 and *y* = 1; so 1 = 2 + 3*b*, *b* = –1/3 and thus the line is *y* = 2 –(1/3)*x*.

1. We first need to find the intercept by solving *y* = *a* + *bx* for *a* with *b* = 4, *x* = –2, *y* = –2; so –2 = *a* +4×–2, *a* = 6 and thus the line is *y* = 6 + 4*x*.
2. First we need to calculate the gradient as being equal to the change in *y* divided by the change in *x* = Δ*y*/Δ*x* = (6-2)/(–2–4) = 4/–6 = –2/3. We can then use either of the two points together with the intercept to obtain the gradient. Hence find the gradient by solving *y* = *a* + *bx* for *a* with *b* =-(2/3), *x* = 4, *y* = 2; so 2 = *a* + –(2/3) × 4, *a* = (3/4) and the line is *y* = (3/4) –(2/3)*x*.

8. (a) *y* = 6*x*, , .

(b) *y* = 3*x*2 + 2, , .

(c) *y* = 4*x*3 + 10, , .

(d) , .

(e) *y* = *x*, , .

(f) *y* = 7, , .

(g) , .

1. *y* = 3ln *x*, , .
2. *y* = ln(3*x*2), , .

(j) ,

,

.

9. All we do here is to apply the standard rule for differentiation for *x* (treating all the terms in *y* as if they were constants) and then separately for *y* (treating all the terms in *x* as if they were constants)

(a) *z* = 10*x*3 + 6*y*2 – 7*y*, ; .

(b) *z* = 10*xy*2 – 6, ; .

(c) *z* = 6*x*, ; .

(d) z = 4, ; .

10. (a) *x*2 – 7*x* – 8 = (*x* – 8)(*x*+1).

(b) 5*x* – 2*x*2 = *x*(5 – 2*x*).

(c) 2*x*2 – *x* – 3 = (2*x* – 3)(*x* + 1).

(d) 6 + 5*x* – 4*x*2 = (2 + *x*)(3 – 4*x*).

(e) 54 – 15*x* – 25*x*2 = (9 + 5*x*)(6 – 5*x*).

11. (a) 53 = 125, ln5 125 = 3.

(b) 112 = 121, ln2 121 = 11.

(c) 64 = 1296, ln4 1296 = 6.

12. (a) ln10 10000 = 4.

(b) ln2 16 = 4.

(c) ln10 0.01 = –2.

(d) ln5 125 = 3.

(e) ln*e* *e*2 = 2.

13. (a) ln5 3125 = 5, 55 = 3125.

(b) ln49 7 = ½, 491/2 = 7.

(c) ln0.5 8 = –3, 0.5-3 = 8.

14. (a) ln 60 = ln(4×3×5) = ln(22×3×5) = 2ln 2 + ln 3 + ln 5.

(b) ln 300 = ln(3×2×2×5×5) = ln(3×22×52) = ln 3 + 2ln 2 + 2ln 5.

15. (a) ln 27 – ln 9 + ln 81 = ln 33 – ln 32 + ln 34 = 3ln 3 – 2ln 3 + 4ln3 = 5ln 3.

(b) ln 8 – ln 4 + ln 32 = ln 23 – ln 22 + ln 25 = 3ln 2 – 2ln 2 + 5ln 2 = 6ln 2.

16. (a) ln *x*4 – ln *x*3 = ln 5*x* – ln 2*x*,  , , *x* = 5/2.

(b) ln (*x*–1) + ln (*x*+1) = 2ln (*x*+2), , (*x*+1)(*x*–1) = (*x*+2)2, *x*2 – 1 = *x*2 + 4*x* + 4, 4*x* = –5, *x* = -5/4.

(c) ln10 *x* = 4, *x* = 104 = 10000.

17. (a) ln 16 = ln (8×2) = ln 8 + ln 2 = ln 8 + ln 81/3 = 4/3 ln 8 = 4/3 × 2.1 = 2.9 (to one decimal place)

(b) ln 64 = ln (8×8) = ln 8 + ln 8 = 2ln 8 = 4.2 (to one decimal place).

(c) ln 4 = ln (8/2) = ln 8 – ln 2 = ln 8 – ln 81/3 = ln 8 – 1/3 ln 8 = 2/3 ln 8 = 2/3 × 2.1 = 1.5 (to one decimal place).

18. (a) 4*x* = 6, *x* = ln4 6 = 1.3 (to one decimal place).

(b) 42*x* = 3, 2*x* = ln4 3 = 2×0.79 = 1.6 (to one decimal place).

(c) 32*x*-1 = 8, (2*x*-1) = ln3 8 = 1.9, *x* = 1.4 (to one decimal place).

19. To find the minimum of a function, we need to differentiate the function and set this first derivative to zero. We would then find the second derivative and ensure that this is above zero to verify that the turning point is indeed a minimum. To find the value of the function at the minimum, we need to plug the value of *x* obtained back into the original formula for *y*

1. *y* = 6*x*2 – 10*x* – 8, , *x* = 5/6. . If *x* = 5/6,

*y* = –12.2 (to one decimal place).

(b) There are two ways that we could solve this problem. The first would be to expand the parentheses so that we have standard terms in *x* (including *x*4, *x*2 and so on) or we could use the rule for differentiating a power of a function, which is

. So for *y* = (6*x*2 – 8)2,

.

If we solve this, we get three solutions: *x*=0, *x*=4/3 and *x*=-4/3. The second derivative is . We need to plug each of the roots *x*=0, *x*=4/3 and *x*=-4/3 into this equation sequentially to identify which corresponds to a minimum. When *x*=0, ; when *x*=4/3, ; and when *x*=-4/3, . So the function has two minima when *x*=4/3 and when *x*=-4/3 and one maximum when *x*=0. When *x*=4/3, *y*=64/9 and when *x*=-4/3, *y*=64/9.

20. There is clearly no shortage of possible examples, but suppose we picked

.

The first thing to note is that the matrices are conformable for multiplication as *A* is 2×2 and *B* is 2×2 so *AB* will also be 2×2

, .

We also have , so that

.

Hence this example has illustrated that (*AB*)-1 =*B*-1*A*-1.

21. (a) For a pair of matrices to be capable of being multiplied together, they must be conformable so that the number of columns of the first matrix is equal to the number of rows of the second. On this basis, the following multiplications are permissible: *AB*, *BA*, *AC*, *DA*, *BC*, *DB*, and *CD*.

(b) To multiply matrices by a scalar (including scalars less than one), we simply multiply each element of that matrix by the scalar. So

, .

(c) Recall that the trace is the sum of the elements on the leading diagonal. So Tr(*A*) = 1 + 4 = 5, Tr(*B*) = –3+4 = 1

 and Tr(*A*+*B*) = –2 + 8 = 6.

Tr(*A*) + Tr(*B*) = 5 + 1 = 6 = Tr(*A*+*B*).

(d) The rank is the number of linearly independent rows or columns in a square matrix. We can see in the case of the matrix *A* that all the columns and rows are linearly independent of one another and hence the matrix is of full rank, 2.

(e) To find the eigenvalues, *λ*, of the matrix *A+B*, we need to solve , where *I* is a 2×2 identity matrix. From part (c) above, .

 , so  and thus (–2–*λ*)(8–*λ*) –(4×–2) = 0. So we have –16 – 8*λ* + 2*λ* + *λ*2 + 8 = 0, *λ*2 – 6*λ* – 8 = 0. This equation does not factorise and thus the quadratic formula can be used, giving the eigenvalues as *λ* = –1.12 and *λ* = 7.12 to two decimal places.

(f) An identity matrix of order 12 will be a square 12×12 matrix having one as each element of the leading diagonal and zero elsewhere. Since the trace is the sum of the elements on this leading diagonal, it will be 1+1+…+1 = 12.

22. (a) .

1. .
2. The inverse of  is .
3. No, since the determinant is 3×2 – 3×2 = 0, the matrix is not of full rank and therefore its inverse does not exist.

23. (a) The first step here is to construct a notation – i.e., a set of symbols which can be used to denote each of the variables. So let f(.) denote a function, *e* denote the US dollar to British pound exchange rate (i.e., the number of dollars per pound), rUS and rUK denote the US and UK interest rates, respectively. Then we could write the relationship as

*e* = f(*rUS*, *rUK*)

We could represent this with an equation for a straight line

*e* = *a* + *b*1*rUS* + *b*2*rUK*

where *a* is the intercept and *b*1, *b*2 are the slope parameters.

(b) Everything else equal, we might expect a rise in US interest rates to cause the US dollar to strengthen and so the number of dollars per pound to fall; on the contrary, a rise in UK interest rates should cause the pound to strengthen relative to the dollar, so the number of dollars per pound to increase. Therefore, we would expect *b*1 < 0 and *b*2 > 0. We probably have no particular expectations about the sign of the intercept term, *a*.

(c) In fact, any values for *b*1 and *b*2 that satisfied b1 = –3*b*2 would do. For example, *b*1 = –0.5; *b*2 = 1.5.

24. Any discontinuous function cannot be differentiated – so any with a drop or sudden jump would not be differentiable. However, even some continuous functions cannot be differentiated, such as the absolute value function, which has a change of direction at *x* = 0, but is strictly viewed as being continuous.